

Analysis to Consumption in Student Canteen Based on High Dimension Lancaster Preference

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Abstract: Lancaster Preference, as a special type of preference, assumes that consumer's utility depends on elements of goods for consumption rather than goods themselves. But according to author's accessible essays, Lancaster limited his case analysis less than 3 elements. This essay extends case analysis of Lancaster preference from low dimension to high dimension theoretically and, based on which, analyzes food consumption in student canteen and explains it well.

1. Introduction

Goods in student canteen are various foods, the elements of which vary from carbohydrate, protein to vitamin, healthy metal elements and others. Students obtain energy and keep healthy through those foods. Specifically, students cannot get enough energy to tackle their studying without enough carbohydrate, and are prone to get sick without enough vitamin. Conversely, as long as having had sufficient and balanced diet, students do not pay too much attention to certain foods. In summary, it is the consumption bundle of elements, rather than foods, that matters in utility maximization of food consumption.

Lancaster preference, a preference different from conventional consumption preference, is researched earliest by Kelvin John Lancaster, a mathematical economist from Australia [1]. This preference has defined goods as combination of more fundamental "elements", which, to some extents, could be regarded as an analogy of production theory [2]. The pity is that Lancaster limited his case analysis in 2 or 3 elements' cases [3]. Consumption in student canteen matches Lancaster's model well, and is a typical multi-elements case, so this essay analyses canteen consumption issue using Lancaster model.

With regard to calculation and discussion of low-dimensional Lancaster framework in a textbook of advanced microeconomics in China [4], this essay extends Lancaster's case analysis to high-dimensional cases based on food consumption with various elements.

2. Basic attributes of lancaster preference

2.1 Basic assumption

Goods in student canteen are various foods, including rice, meat, vegetables and so on. Those foods are combined with some elements, including carbohydrate, protein, vitamin, healthy metal elements and others.

Assume that $s = (s^1, s^2, \dots, s^r)$ refers to all the elements contained in all available food in canteen, and $s^i \geq 0 (i = 1, 2, \dots, r)$ because element quantity is always nonnegative. If there are k kinds of foods, consumption bundle is $x = (x_1, x_2, \dots, x_k)$. For one unit of food j , elements contained are $a_j = (a_j^1, a_j^2, \dots, a_j^r)$. For one element i , its quantities in all kinds of foods are $a^i = (a_1^i, a_2^i, \dots, a_k^i)$. So consumption bundle x contains elements:

$$s^i = a_1^i x_1 + a_2^i x_2 + \dots + a_k^i x_k = a^i x^T, i = 1, 2, \dots, r$$

Assume that preference of consumers in canteen is well-behaved (continuous, strictly and monotonically increasing, strictly convex), and that consumers only care about elements s , then s determines consumers' utility. Under price vector $p = (p_1, p_2, \dots, p_k)$ and budget restriction m , utility maximization means:

$$\begin{aligned} & \max_{s,x} u(s) \\ & \text{s.t. } px^T \leq m \\ & s^i = a^i x^T, i = 1, 2 \dots r \end{aligned}$$

2.2 Restriction set and elements' implicit price

Let's consider restriction set at first. Because there are r types of elements, restriction set should be in a r -dimension space with r coordinate axis. Assumes that original point is O and the element i correspond to the axis s^i . Because there are k kinds of foods, so there are k half-line. On half-line OG_j , food j 's coordinate is $a_j t = (a_j^1 t, a_j^2 t, \dots, a_j^r t)$, $t \geq 0$. If consumer spend the total budget m on food j , consumption of food j is $\frac{m}{p_j}$, and corresponding element consumption point on half-line OG_j is $A_j = a_j \frac{m}{p_j} = (a_j^1 \frac{m}{p_j}, a_j^2 \frac{m}{p_j}, \dots, a_j^r \frac{m}{p_j})$. In the r -dimension space, the original point and points A_1, A_2, \dots, A_k construct an ω -dimension sub-space Ω , $\omega \in \{1, 2, \dots, r\}$. All points in Ω are affordable.

Due to monotonic preference, the consumer must spend out the budget m . So the restrictions are below:

$$\begin{aligned} p_1 x_1 + p_2 x_2 + \dots + p_k x_k &= m \\ a_1^1 x_1 + a_2^1 x_2 + \dots + a_k^1 x_k &= s^1 \\ a_1^2 x_1 + a_2^2 x_2 + \dots + a_k^2 x_k &= s^2 \\ &\dots \\ a_1^r x_1 + a_2^r x_2 + \dots + a_k^r x_k &= s^r \end{aligned}$$

The latter r equations could be rewritten as $Ax^T = s^T$, which is assumed to be solvable. Mathematically, it means that $r(A) = r(A|s^T)$ and the solution is $x^{*T} = Bs^T$. After substituting x^T in $px^T = m$ with x^{*T} , it generates $pBs^T = m$. Defines r -dimension vector $\bar{P} = pB = (\bar{p}^1, \bar{p}^2, \dots, \bar{p}^r)$, then $(\bar{p}^1, \bar{p}^2, \dots, \bar{p}^r)$ determines implicit price of (s^1, s^2, \dots, s^r) , and: .

$$A\bar{P}^T = AB^T p^T = p^T$$

Here $AB^T = E$ is used, because $Ax^{*T} = AB^T s^T = s^T$. The left side of equation $A\bar{P}^T = p^T$ is the value calculated by implicit price of elements in food consumption bundle $x = (x_1, x_2, \dots, x_k)$, and on the right side is the food price of $x = (x_1, x_2, \dots, x_k)$.

2.3 Efficient consumption frontier

Efficient consumption frontier represent the set of elements bundle in every available relative proportion when all the budget m is spent out. In another word, it is the $(\omega-1)$ -dimension manifold $\bar{\Omega}$ which is the furthest part from original point on ω -dimension sub-space Ω . Elements on efficient consumption frontier have certain substitutive rate according to equal marginal:

$$\frac{\partial u / \partial s^i}{\pi^i} = \frac{\partial u / \partial s^j}{\pi^j}, \forall i, j \in \{1, 2, \dots, r\}, i \neq j$$

In r -dimension space, optimal consumption s^* is the tangent point of efficient consumption frontier $\bar{\Omega}$ and indifference $(\omega-1)$ -dimension manifold, and the coordinate of s^* , $(s^{1*}, s^{2*}, \dots, s^{r*})$, represent optimal consumption of all elements. In most cases, s^* happens only when multiple goods are consumed, when s^* is an inner point in a $(\omega-1)$ -dimension smooth manifold $\tilde{\Omega}$ which is a subset of $\bar{\Omega}$ and surrounded by ω points A_i and line segments $A_i A_j$ ($i, j \in \{1, 2, \dots, r\}, i \neq j$) that connect those points. Then, projects line segment Os^* to ω line segments OA_i , and assumes that rate between the projection and OA_i is t_i , then $\sum t_i = 1$. Here t_i is also allocation rate of budget m to the good i , which means in every one unit of budget, consumer spent t_i on the good i . However, in peculiar cases, s^* is one of A_i rather than an inner point, which means optimal consumption is to only buy the good i .

According to above discussion, efficient consumption law could be proved easily. This law indicates that, if there are r types of elements and $k > r$ types of goods, then the efficient consumption

frontier contains not more than r types of goods. s^* is an inner point in a $(\omega-1)$ -dimension smooth manifold $\tilde{\Omega}$ in Ω , an ω -dimension subspace, and the projection of s^* is on ω line segments OA_i , which means s^* corresponds to ω types of goods. Because Ω is subspace of r -dimension space, obviously $\omega \leq r$. As long as consumer consumes ω types of goods, so consumes not more than r types of goods even if there are totally $k > r$ types of goods.

3. Specific practice of Lancaster preference in student canteen consumption

3.1 Description to canteen consumption using Lancaster preference

Lancaster preference could depict canteen consumption. Consumed cuisine are consumption bundle $x = (x_1, x_2, \dots, x_k)$, the price of which is $p = (p_1, p_2, \dots, p_k)$. Contained elements are element consumption bundle $s = (s^1, s^2, \dots, s^r)$, the implicit price of which is $\bar{P} = (\bar{p}^1, \bar{p}^2, \dots, \bar{p}^r)$.

3.2 Explanation of elements' utility using law of equal marginal

According to commonsense, cuisine in student canteen could be categorized in 3 groups: carbohydrates, vegetables and meats, which is ordered from low price to high price. They respectively contain 3 groups of elements: carbohydrate; cellulose, vitamin and plant protein; animal protein.

3 groups of elements are respectively main ingredients in 3 groups of foods, indicating that ranking of food price P is very close to ranking of elements' implicit price \bar{P} . According to law of equal marginal, consumption of certain type of element proportionate with implicit price. As a result, \bar{P} could be used to estimate marginal utilities of corresponding elements. For example, rice and steamed bread are typical carbohydrate food, so their low prices reveal low marginal utility of carbohydrate. Various of green vegetables are typical foods that contain cellulose, vitamin and plant protein, so their middle-level prices reveal middle-level marginal utility of those elements. Meats contain animal protein the most, and their high prices reveal high marginal utility of animal protein.

Nevertheless, as marginal utility does not necessarily equal to total utility brought by one element, we could not dictate either that carbohydrate brings the least utility or that meat brings the highest utility. It should be analyzed comprehensively. Regardless of low marginal utility of carbohydrate, the reason is its high consumption quantity and law of decreasing marginal utility. Because of similar reason, low consumption quantity of meat indicates meat's high marginal utility. In summary, total utilities of 3 groups of elements do not differ so largely as their marginal utilities.

3.3 Explanation of foods' consumption using efficient consumption law

According to commonsense, one consumption for one consumer in a canteen contains about 3 or 4 types of foods in most cases. Tens of elements could be related even though they are all in 3 or 4 types of foods.

Consist with 3 roughly categorized groups of foods, all types of elements could also be categorized into 3 corresponding groups, which include carbohydrate elements, vegetable elements and meat elements. Under this framework, tens of types of elements are simplified to 3 groups, which consists with efficient consumption law, because consumers consume not more than 3 types of foods.

In add to efficient consumption law, another key reason is minimal cuisine allocation in practice and maximal quantity of consumption for one consumer. Because consumer have to order not less than a certain quantity of a food, consumers can never consume tens of types of foods for one time, otherwise it would be too much. Heterogeneous maximal consumption for different consumers also explain heterogeneous number of food types in one consumption. For those who cannot eat much, ordering 2 types of foods is enough, but ordering 4 types is necessary for those who eat much. Nevertheless, *ceteris paribus*, efficient consumption law still works well.

3.4 Limitation of Lancaster preference

Although Lancaster preference could explain consumption in student canteen well, its limitation

is conspicuous. In add to maximal consumption mentioned in part 3.3, other factors include budget, taste, fat and so on.

3.4.1 Budget

Lancaster preference assumes that budget m is constant and consumers decide consumption under this restriction. However, in canteen cases, budget m is always not limited for one consumption. Conversely, m vary with consumption bundle, which means that consumer shall adjust budget. In special cases, m is actually restricted because of consumer's poverty.

3.4.2 Taste

Utility in canteen consumer do not depend only on ingredient elements in foods, but also on their taste. Even though elements are not various, a tasty food encourage consumption. Conversely, food with terrible taste is not likely to be consumed much even if it contains much ingredients.

3.4.3 Elements with minus utility

Lancaster preference does not consider marginal utility that decrease to minus value, which limit its universality [5]. Some ingredient elements should not be taken too much, otherwise it would be harmful, such as fat. Especially for those who need to keep a slim body, such as young female, such a consider is likely to be unignorable.

3.4.4 Irrationality

Conventional consumption theory, assuming consumers pursue maximal utility under budget restriction, is based on complete rationality of consumers. However, with the arising of behavioral economics, irrationality that could affect consuming behavior has been emphasized by academy [6]. Research in breakfast pricing indicates that irrational factors are likely to affect pricing, then cause seemingly unreasonable phenomena [7]. Lancaster preference is conventional, and don not consider psychological factors. Some emotional factors, such as impulsive consumption, is not considered by Lancaster preference.

4. Conclusion

This essay uses Lancaster preference to analyze consumption in student canteen and provides a relatively sound explanation. However, it does not explain everything because of its limitation. Analysis in this essay could be extended further, such as empirical analysis using price and consumption data in canteen to support this essay.

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